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EFFECTS OF PROBLEM DIFFICULTY AND REINFORCER QUALITY ON TIME ALLOCATED TO CONCURRENT ARITHMETIC PROBLEMS

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Students with learning difficulties participated in two studies that analyzed the effects of problem difficulty and reinforcer quality upon time allocated to two sets of arithmetic problems reinforced according to a concurrent variable-interval 30-s variable-interval 120-s schedule. In Study 1, high- and low-difficulty arithmetic problems were systematically combined with rich and lean concurrent schedules (nickels used as reinforcers) across conditions using a single-subject design. The pairing of the high-difficulty problems with the richer schedule failed to offset time allocated to that alternative. Study 2 investigated the interactive effects of problem difficulty and reinforcer quality (nickels vs. program money) upon time allocation to arithmetic problems maintained by the concurrent schedules of reinforcement. Unlike problem difficulty, the pairing of the lesser quality reinforcer (program money) with the richer schedule reduced the time allocated to that alternative. The magnitude of this effect was greatest when combined with the low-difficulty problems. These studies have important implications for a matching law analysis of asymmetrical reinforcement variables that influence time allocation.

DESCRIPTORS: problem difficulty, reinforcer quality, matching law, concurrent schedules

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The matching law (Herrnstein, 1961, 1970) states that the relative rate of responding on concurrently available alternatives is equal to the relative rate of reinforcement obtained from these alternatives. Numerous basic (de Villiers, 1977; Pierce & Epling, 1983) and applied (Conger & Killeen, 1974; Mace, Neef, Shade, & Mauro, 1994; Martens & Houk, 1989; Martens, Lochner, & Kelly, 1992; Neef, Mace, & Shade, 1993; Neef, Mace, Shea, & Shade, 1992) research studies have shown that the matching relation describes how human behavior is allocated in choice situations. However, several authors have cautioned that important differences in laboratory and natural human environments may preclude direct extensions of the matching law to applied matters (e.g., Fuqua, 1984; Mace et al., 1994). For example, when human choice involves asymmetrical alternatives with differences in reinforcer quality, reinforcer delay, or the response effort needed to produce reinforcement, then relative reinforcement rate alone may be insufficient to predict response al-

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location (Davison & McCarthy, 1988; McDowell, 1989).

In two previous studies, we examined the effects of arranging qualitatively different reinforcers for different response alternatives (Neef et al., 1992) and varying the delay to reinforcement across alternatives (Neef et al., 1993) on choice behavior. Baseline conditions for both studies involved concurrent variable-interval variable-interval (VI VI) schedules of reinforcement for adolescents' completion of two stacks of arithmetic problems, while other response and reinforcement parameters were held constant. Following baseline, reinforcers of unequal quality (Neef et al., 1992) or unequal delays to reinforcement (Neef et al., 1993) were arranged for the two stacks of problems. Subjects in both studies showed strong preferences for the higher quality reinforcer and shorter delays to reinforcement. These studies provide evidence for the importance of considering asymmetries in the reinforcement parameters correlated with response alternatives when pursuing matching law accounts of human choice.

Another variable that may influence allocation between concurrent VI VI schedules is the response difficulty required to perform each task. Several studies, without a specific focus on the matching law, have shown that comparatively difficult tasks or those that require greater effort to perform are associated with lower levels of task engagement, accuracy, and compliance (Cooper, Wacker, Sasso, Reimers, & Donn, 1990; Friman, Finney, Rapoff, & Christophersen, 1985; Horner & Day, 1991; Weeks & Gaylord-Ross, 1981). Similarly, rates of maladaptive behavior have been inversely related to the presence or absence of task demands (Carr & Durand, 1985; Carr & Newsom, 1985; Carr, Newsom, & Binkoff, 1980), novel versus familiar tasks (Mace, Browder, & Lin, 1987), tasks with greater response requirements (Cooper et al., 1990; Mace et al., 1987), traditional task instruction versus errorless instruction (Weeks & Gaylord-Ross, 1981), and the availability of assistance to perform tasks (Horner & Day, 1991).

Although applied studies have shown socially relevant human behavior to be sensitive to concurrent task difficulty, results of basic matching law studies on this topic have been mixed. Davison and Ferguson (1978) scheduled concurrent VI VI reinforcement for pigeons for which the response alternatives had different topographical requirements. Key pecking, a comparatively low-effort response, was juxtaposed with lever pressing, a response that is difficult for pigeons to emit. The pigeons showed a strong bias for the key-peck response that was independent of the rates of reinforcement derived from the two alternatives. In contrast, differences in physical force requirements across manipulanda have not resulted in response bias. Chung (1965) and Hunter and Davison (1982) parametrically varied the force required to execute two response alternatives subject to concurrent VI VI schedules of reinforcement. Neither study found that increasing the physical force required to operate one manipulandum resulted in bias for the alternative response having a lesser force requirement.

The present research had two major goals. Using procedures similar to Neef et al. (1992, 1993), Study 1 examined whether differences in the difficulty of arithmetic problems would alter allocation patterns established in a baseline condition of concurrent VI VI reinforcement. Although several applied studies have shown response difficulty to be an important variable affecting performance, the mixed findings from basic research on the matching law raised questions of whether problem difficulty would mediate the effects of reinforcement rate alone. The goal of Study 2 was to assess whether differences in reinforcer quality would interact with differences in problem difficulty to produce an overriding preference for alternatives associated with comparatively low problem difficulty and high-quality reinforcement.

STUDY 1

METHOD

Subjects and Setting

Two students in a special education program for youth with severe emotional disturbance or behavior disorders and learning difficulties served as subjects. The students' teachers referred them for treatment because of their need for assistance in completing instructional tasks and for practice in arithmetic skills. They were enrolled in the study following the receipt of informed consent by the participants and their guardians. Both students had been subjects in a previous study on the effects of reinforcer rate and reinforcer quality on time allocation (Neef et al., 1993). The 2 students, Ivana and Joyce, were 18 and 14 years of age, respectively. Both were diagnosed as functioning within the borderline range of cognitive ability. Their math scores on the California Achievement Test yielded a 7.2 and 7.0 grade level, respectively. Sessions were conducted in a small office at the school, with the experimenter seated across from the participant at a table.

Experimental Conditions and Procedures

The experimental task was the same as that described in Neef et al. (1993). Two 10-min sessions were conducted per day, 3 days per week, for each participant. During each session, two stacks of problems, printed on yellow and goldenrod index cards, were placed on the table in front of the student. The arithmetic problems were chosen on the basis of the recommendation of the classroom teacher and on standardized test scores (California Achievement Test) indicating the student's current level of math skill. The types of problems in each stack varied according to the experimental condition in effect (described below). The student was given a standard instruction, "You can earn nickels doing these math problems. You may work on either stack of problems as you choose. You may start when I say 'begin.'"

Correct responses to problems on yellow and goldenrod cards were reinforced with nickels on concurrent VI 30-s and VI 120-s schedules, respectively. The nickels were deposited in a transparent plastic cup the same color as, and directly behind, its respective stack of problems. Reinforcement was delivered contingent on the first correctly completed problem after the reinforcement interval had elapsed (signaled to the experimenter by audiotape through an earphone). Following an incorrect response, the experimenter marked the card with an X.

As in Neef et al. (1993), a prebaseline condition was conducted with Joyce in which a countdown kitchen timer that indicated the time remaining in the reinforcement interval was placed behind each stack of problems. The timers were successively introduced and withdrawn (and ultimately faded) over blocks of 10 to 12 sessions until Joyce consistently allocated her behavior in patterns predicted by the schedules without the timers. This condition was in effect for 57 sessions, after which the experimental conditions were initiated.

Equal difficulty. Performance was assessed under two concurrent VI 30-s VI 120-s schedules. Low-difficulty or high-difficulty problems were used alternately across sessions and served as the stimuli for both schedules. Thus, problem difficulty was held constant across the two sets of arithmetic problems within each set of concurrent schedules.

For Ivana, low-difficulty problems consisted of two-digit by one-digit multiplication problems. High-difficulty problems changed across successive replications of this condition to minimize the effects of practice on relative difficulty. Initially, high-difficulty problems consisted of three-digit numbers divided by 3 through 12. Across subsequent replications of the equal-difficulty condition, high-difficulty problems consisted of four-digit numbers divided by 6 through 9, 11 through 15, and 16 through 19, respectively. For Joyce, two-digit plus two-digit addition problems served as the low-difficulty problems. High-difficulty problems across replications of this condition consisted of three-digit by one-digit multiplication, four-digit by one-digit multiplication, and division of single-digit fractions, respectively.

Neef, Shade, and Miller (1994) investigated the interactive effects of reinforcement rate, response effort, and reinforcer delay and established a precedence for the operationalizing of problem difficulty as differences in fluency and accuracy. The present operationalizing of problem difficulty aligns with accepted curricula practices in which certain arithmetic operations are taught prior to others (e.g., multiplication prior to division). The criterion for defining problem difficulty in the present study (as well as in Study 2) was different across subjects. In some instances, it depended on the operation involved, whereas in other instances it depended on the number of digits or number of steps involved in a problem. The decision to vary the type of operation involved or the number of digits or steps was guided by the recommendations of the classroom teacher (see the Appendix for fluency and accuracy measures obtained during equal-difficulty conditions).

Unequal Difficulty 1. Schedules of reinforcement for the two response options (i.e., for performance of problems on yellow vs. goldenrod cards) were identical to the equal-difficulty condition, but accurate responses to the low-difficulty problems (on goldenrod cards) were reinforced on the VI 120-s (lean) schedule, and solutions to the high-difficulty problems (on yellow cards) were reinforced on the VI 30-s (rich) schedule. The low- and high-difficulty problems were the same as those in the preceding equal-difficulty condition.

Unequal Difficulty 2. Procedures were identical to the Unequal Difficulty 1 condition except that the color of the problem cards (yellow vs. goldenrod) associated with the different schedules of reinforcement was reversed to determine whether responding was controlled exclusively by the schedule of reinforcement.

Experimental Design

A reversal design was used for both subjects. Table 1 provides an overview of the discriminative stimuli (index card color), reinforcement schedules, and problem difficulty associated with the left and right stacks of arithmetic problems for the conditions of Study 1. For Ivana, the arrangement consisted of A1-B1-A2-B2-A3-B3-A3-A4-B4-C4-B4-C4-B4, in which A, B, and C represent the equal-difficulty, Unequal Difficulty 1, and Unequal Difficulty 2 conditions, respectively, and numbers represent the sets of math problems associated with successively greater differences in difficulty between the two response options. For Joyce, the sequence of experimental conditions was A1-B1-A2-B2-A3-B3-C3-B3-C3-B3.

Data Collection and Interobserver Agreement

Data collectors recorded time allocated to each of the response alternatives within continuous 1-min intervals using two stopwatches. Time allocation was chosen as the dependent measure because it more directly reflects reinforcement schedule control. The respective stopwatch measured the duration of a student's visual orientation toward a problem from a particular stack. A second observer collected interobserver agreement data on 32% of the sessions across conditions. An agreement was defined as both observers recording the same duration within a 1-min

Table 1
Discriminative Stimuli (Index Card Color), Reinforcement Schedule, and Problem Difficulty Associated with the Left or Right Stack of Arithmetic Problems for the Three Conditions in Study 1.

	Left stack of problems			Right stack of problems		
Condition	Color	Schedule	Difficulty	Color	Schedule	Difficulty
Equal difficulty (A)	Yellow	Rich	Low	Goldenrod	Lean	Low
	Yellow	Rich	High	Goldenrod	Lean	High
Unequal Difficulty 1 (B)	Yellow	Rich	High	Goldenrod	Lean	Low
Unequal Difficulty 2 (C)	Yellow	Lean	Low	Goldenrod	Rich	High

Note. The conditions shown across the two rows for equal difficulty (A) were alternated across sessions.

interval (±2 s). Interobserver agreement measures were obtained for 41% and 30% of the sessions for Ivana and Joyce, respectively, with mean agreement scores of 91.7% (range, 70% to 100%) and 100%. The number of reinforcers delivered was also recorded, and the exact interobserver agreement on this measure was 98.9% (range, 80% to 100%) for Ivana, and 100% for Joyce.

Results and Discussion

Figures 1 and 2 show the extent of time-allocation matching across equal- and unequal-difficulty conditions for Ivana and Joyce, respectively. Time-allocation matching is represented by the relationship between the percentage of time engaged in problems on the yellow cards (subject to the VI 30-s reinforcement schedule) and the percentage of obtained reinforcement on that schedule. The former measure is calculated as $T_1/(T_1 + T_2) \times 100$, where T_1 and T_2 are the total amount of time spent on the task alternative subject to the VI 30-s and VI 120-s schedules, respectively. The latter measure is calculated as $r_1/(r_1 + r_2) \times 100$, where r_1 and r_2 represent the obtained rates of reinforcement on those alternatives. For both students, a comparison of the A and B phases showed that the percentage of time allocated to each response alternative closely corresponded to the amount of obtained reinforcement, regardless of whether the responses subject to the VI 30-s schedule were of greater or equal difficulty relative to those of the VI 120-s schedule.

For Ivana, the mean difference between the percentage of time allocation and obtained reinforcement across the first four phases (alternations of equal- and unequal-difficulty conditions) was 4.4%, 4.2%, 5.1%, and 3.9%, respectively. The difference increased during the subsequent equal-difficulty condition ($M = 13\%$), but approached previous levels during the following unequal-difficulty condition ($M = 7.4\%$) and reinstatement of the equal-difficulty conditions ($M = 4\%$). When the color of the math problem cards (yellow vs. goldenrod) associated with the VI 30-s and VI 120-s schedules was reversed, there were immediate changes in the percentage of time allocation to problems on the yellow cards corresponding to the reinforcement schedule. The mean differences between percentage of time allocation and obtained reinforcement across successive alternations of Unequal Difficulty 1 and Unequal Difficulty 2 conditions were 8.2%, 6.0%, 8.7%, 8.1%, and 9.7%, respectively.

During the first equal-difficulty condition, the proportion of time Joyce allocated to the VI 30-s schedule was somewhat less than the proportion of reinforcement she

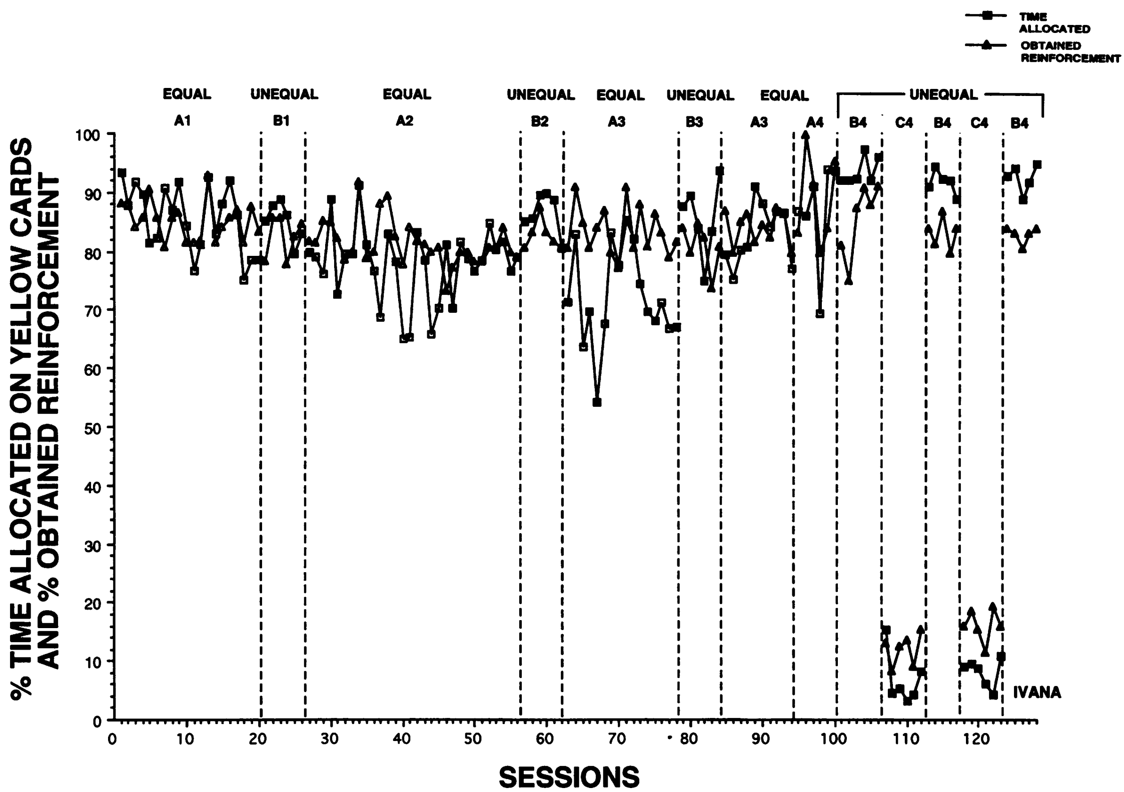

Figure 1. Ivana's performance on concurrent VI 30-s VI 120-s schedules of reinforcement across arithmetic problems of equal and unequal difficulty. Data are presented as percentage of time allocated (squares) and reinforcement obtained (triangles) from performing arithmetic problems on the yellow cards. The open and closed symbols during the equal-difficulty condition (only) represent the low- and high-difficulty problems that alternated across sessions, respectively.

obtained from that schedule (mean difference = 14.8%). This pattern continued during the subsequent equal and Unequal Difficulty 1 conditions, although, overall, there was high correspondence between time allocation and obtained reinforcement. The mean differences across conditions were 8.6%, 8.1%, 9.4%, 7.4%, and 2.7%, respectively. As with Ivana, reversing the color of the stimuli associated with the VI 30-s and VI 120-s schedules produced corresponding reversals in the time allocated to those alternatives and in proportion to obtained reinforcement. The mean difference across the final three phases was 4.4%, 7.7%, and 6.9%, respectively.

Results for both subjects were qualitatively similar. When choices were symmetrical according to problem difficulty, relative time allocated to the two stacks of arithmetic problems approximated the proportion of reinforcement obtained from the alternatives. However, this correspondence continued despite successive increases in the complexity of the arithmetic problems assigned to the VI 30-s schedule. Study 2 was designed to assess possible interactions between reinforcer quality and problem difficulty in a similar study design.

STUDY 2

Method

Subjects and Setting

Ivana (who participated in Study 1) and Nicole, another student in the program, par-

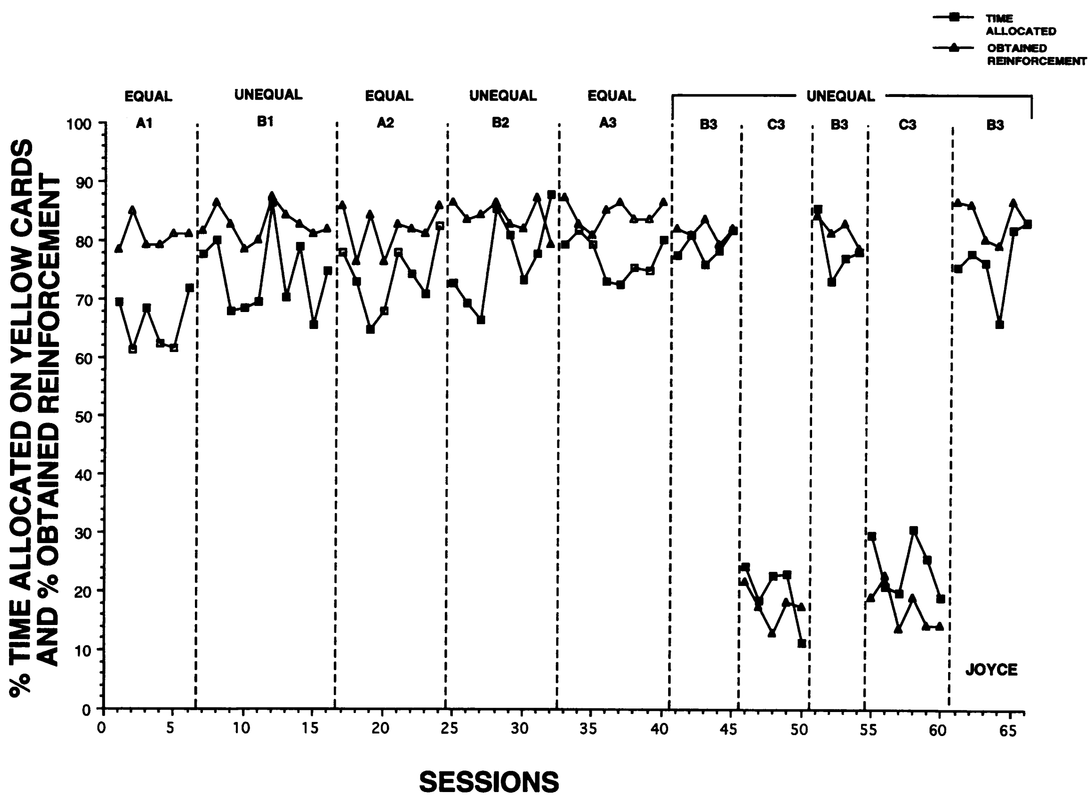

Figure 2. Joyce's performance on concurrent VI 30-s VI 120-s schedules of reinforcement across arithmetic problems of equal and unequal difficulty. Data are presented as percentage of time allocated (squares) and reinforcement obtained (triangles) from performing arithmetic problems on the yellow cards. The open and closed symbols during the equal-difficulty condition (only) represent the low- and high-difficulty problems that alternated across sessions, respectively.

ticipated in Study 2. Nicole, aged 13, had diagnoses of posttraumatic stress disorder, dysthymia, and atypical pervasive developmental disorder. She functioned within the average range of intelligence. Her psychoeducational test scores on math were at the 3.2 grade level. The setting was the same as for Study 1.

Procedures and Independent Variables

Nicole participated in a prebaseline condition, as previously described, in which six sessions without the use of timers were followed by six sessions with timers. Experimental conditions were then initiated. Data-collection methods and experimental procedures were identical to those described in Study 1, with the exception that the reinforcer as well as problem difficulty associated with each concurrent schedule varied according to the experimental condition in effect (described below).

Reinforcer rate. As in Study 1, correct responses to problems on yellow and goldenrod cards were reinforced on concurrent VI 30-s and VI 120-s schedules, respectively.

Reinforcer quality. For both students, nickels were designated as the high-quality reinforcer, and their equivalent in program money (exchangeable in the school's token economy system for community outings, privileges or special events, and items from the school store) served as the low-quality reinforcer. (High- and low-quality designations were made on the basis of asking the student at the start of each session whether

Table 2
Discriminative Stimuli (Index Card Color), Reinforcement Schedule, Problem Difficulty, and Reinforcer Quality Associated with the Left or Right Stack of Arithmetic Problems for the Conditions Used in Study 2.

	Left stack of problems				Right stack of problems			
Condition	Color	Schedule	Difficulty	Quality	Color	Schedule	Difficulty	Quality
Equal difficulty								
Equal quality	Yellow	Rich	High	Low	Goldenrod	Lean	High	Low
	Yellow	Rich	High	High	Goldenrod	Lean	High	High
Unequal Quality 1	Yellow	Rich	High	Low	Goldenrod	Lean	High	High
Unequal Quality 2	Yellow	Rich	High	High	Goldenrod	Lean	High	Low
Unequal difficulty								
Unequal Quality 1	Yellow	Rich	Low	Low	Goldenrod	Lean	High	High
Unequal quality	Yellow	Rich	High	High	Goldenrod	Lean	Low	Low

Note. The conditions shown across the two rows for equal quality were alternated across sessions.

she preferred to work for nickels or program money; Neef et al., 1992.)

Problem difficulty. For Ivana, low-difficulty (two-digit by one-digit multiplication) and high-difficulty problems (four-digit numbers divided by 16, 17, 18, or 19) were the same as in the final unequal-difficulty condition in Study 1. For Nicole, low-difficulty problems consisted of three-digit plus three-digit addition without a carrying operation, and high-difficulty problems consisted of three-digit minus three-digit subtraction with a borrowing operation.

Experimental Conditions

Equal difficulty/equal quality. Two baseline conditions were alternated across sessions to establish the student's sensitivity to varying reinforcer rates while levels of reinforcer quality and problem difficulty were held constant. During one condition, high-difficulty problems and the low-quality reinforcer were used for both VI schedules. The other condition was identical except that the high-quality reinforcer was used for both VI schedules.

Equal Difficulty/Unequal Quality 1. Both response alternatives consisted of high-difficulty problems, but the low-quality reinforcer was used for the set of problems associated with the VI 30-s schedule, and the high-quality reinforcer was used for the set of problems associated with the VI 120-s schedule.

Equal Difficulty/Unequal Quality 2. Conditions were identical to the previous phase except that the schedule-quality pairings were reversed (i.e., the high-quality reinforcer was paired with the VI 30-s schedule and vice versa).

Unequal Difficulty/Unequal Quality 1. The response alternatives consisted of low-difficulty problems and low-quality reinforcers on the VI 30-s schedule and high-difficulty problems and high-quality reinforcers on the VI 120-s schedule.

Unequal Difficulty/Unequal Quality 2. Conditions were identical to the previous phase except that the problem difficulty–reinforcer quality pairings with the VI schedules were reversed (i.e., the high-difficulty problems and high-quality reinforcers were paired with the VI 30-s schedule and vice versa).

Experimental Design

Table 2 provides an overview of the discriminative stimuli (index card color), reinforcement schedules, problem difficulty, and reinforcer quality associated with the left and right stacks of arithmetic problems for the conditions of Study 2. The reversal of con-

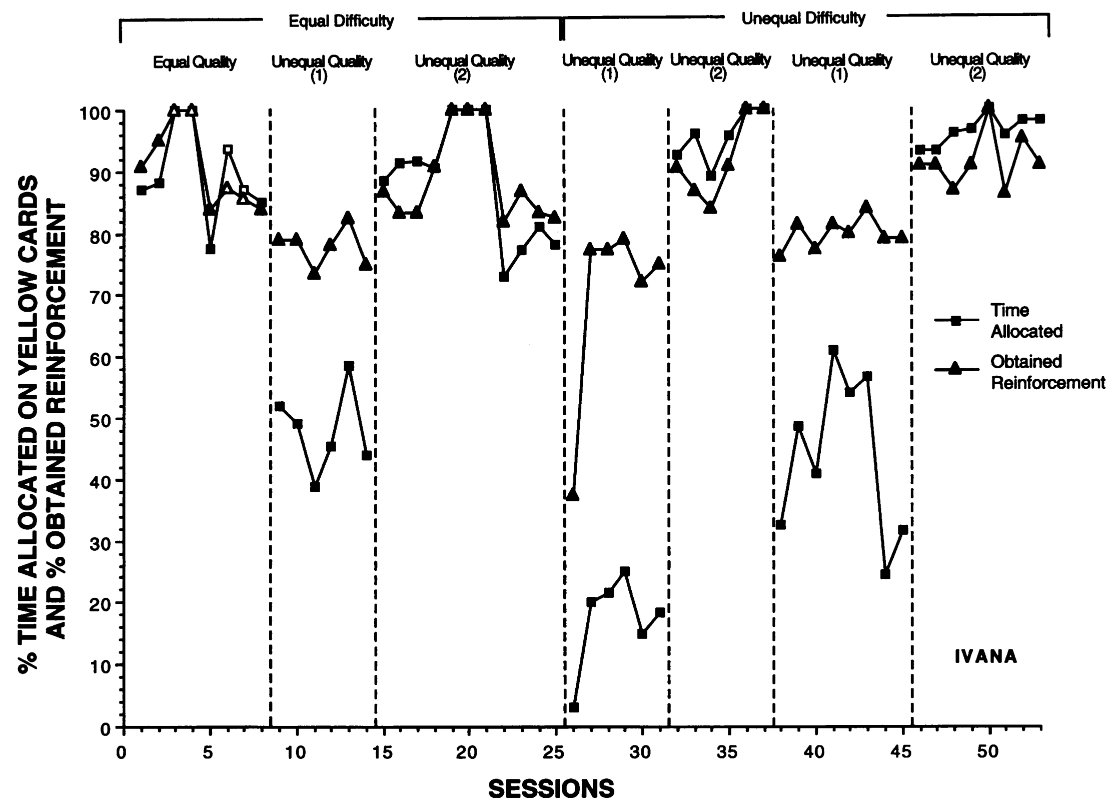

Figure 3. Ivana's arithmetic performance on concurrent VI 30-s VI 120-s schedules of reinforcement across different combinations of equal and unequal difficulty and reinforcer quality. Data are presented as percentage of time allocated (squares) and reinforcement obtained (triangles) from performing arithmetic problems on the yellow cards. The open and closed symbols during the equal-quality condition (only) represent the high- and low-quality reinforcers for both alternatives, respectively.

tingencies across the concurrent schedules in the two equal-difficulty/unequal-quality conditions and in the two unequal-difficulty/unequal-quality conditions provides a replication of the functional relation. In addition, the two unequal-difficulty/unequal-quality conditions were replicated using a reversal design.

Data Collection and Interobserver Agreement

Data collection and interobserver agreement procedures were the same as those used in Study 1. Interobserver agreement was assessed on 28% and 31% of the sessions for Ivana and Nicole, respectively. Mean agreement scores on time allocation were 91.3% for Ivana (range, 80% to 100%) and 99.2% for Nicole (range, 80% to 100%). Mean agreement scores for reinforcer delivery were 98.7% for Ivana (range, 70% to 100%) and 100% for Nicole.

Results and Discussion

Figures 3 and 4 show the relationship between the percentage of time engaged in problems on the yellow cards (subject to the VI 30-s reinforcement schedule) and the percentage of obtained reinforcement on that schedule, across experimental conditions for Ivana and Nicole, respectively. With both students, the percentage of time allocated to each response alternative closely corresponded to the amount of obtained reinforcement when response difficulty and reinforcer quality were the same across the two response alternatives. For Ivana, the mean

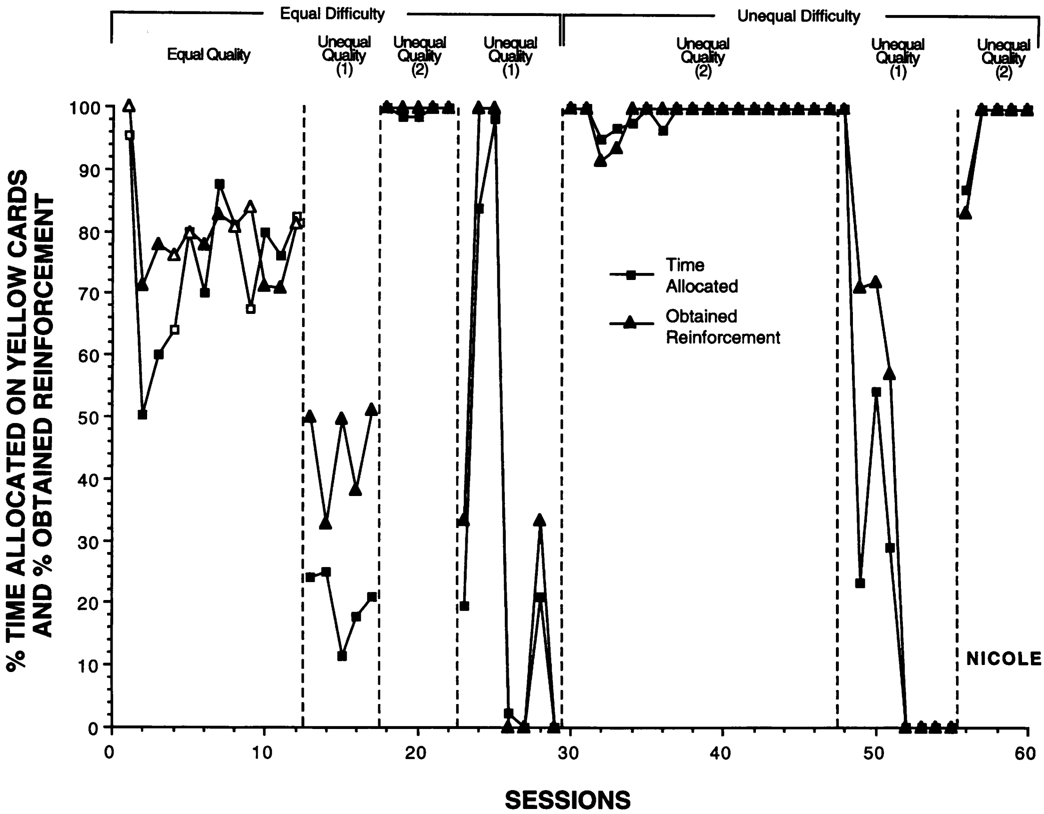

Figure 4. Nicole's arithmetic performance on concurrent VI 30-s VI 120-s schedules of reinforcement across different combinations of equal and unequal difficulty and reinforcer quality. Data are presented as percentage of time allocated (squares) and reinforcement obtained (triangles) from performing arithmetic problems on the yellow cards. The open and closed symbols during the equal-quality condition (only) represent the low- and high-quality reinforcers for both alternatives, respectively.

difference between the percentage of time allocated to the yellow card alternative (M = 89.9%) and the percentage of reinforcement obtained on that alternative (M = 90.9%) was 3.2%. For Nicole, the mean percentages of time allocated to and reinforcement obtained from that alternative, respectively, were 76% and 78.8%, with a mean difference of 7.9%.

For both students, reinforcer quality overrode the effects of rate of reinforcement during the first equal-difficulty/unequal-quality condition, replicating the results of Neef et al. (1992). The mean percentage of time allocated to the yellow card alternative (which, in this condition, was associated with the less preferred reinforcer) decreased to 48% for Ivana and to 19.7% for Nicole. The mean difference between the percentage of time allocation and obtained reinforcement increased to 29.9% and 25.5% for Ivana and Nicole, respectively. When these contingencies were reversed in the second equal-difficulty/unequal-quality condition, there was a corresponding increase in the percentage of time allocated to the yellow card alternative to 88.4% for Ivana and 99.6% for Nicole. The mean difference between time allocation and obtained reinforcement was 3.9% for Ivana and 0.4% for Nicole.

Reinforcer quality also overrode the combined effects of rate of reinforcement and problem difficulty. In the first unequal-difficulty/unequal-quality condition, both students allocated a lower percentage of time to the response alternative associated with the least difficulty and a higher rate of reinforcement but a less preferred reinforcer (*M* = 17.2% for Ivana and 32.2% for Nicole). When these contingencies were reversed in the second unequal-difficulty/unequal-quality condition, both students again allocated the majority of time to the response alternative associated with the greater difficulty but more preferred reinforcer (*M* = 95.5% and 99.2%, respectively).

Subsequent replications of the two unequal-difficulty/unequal-quality conditions produced similar effects. The mean percentages of time allocated to the yellow card alternative during these two conditions were 43.7% and 97% for Ivana and 25.8% and 97.4% for Nicole. The mean differences between time allocation and obtained reinforcement during these conditions were 36.1% and 5.5% for Ivana and 11.8% and 0.8% for Nicole. The effects of reinforcer quality clearly overrode those of rate of reinforcement and problem difficulty (see the last four phases of Figure 3).

GENERAL DISCUSSION

Response alternatives in natural human environments often consist of asymmetrical choices. Alternatives may differ in the rate and quality of reinforcement they produce as well as in the difficulty required to execute the response. In Study 1, we arranged baseline rates of reinforcement to be four times greater for one of two stacks of arithmetic problems while the type of arithmetic problem and, presumably, the difficulty required to solve the problem were held constant. During baseline, both participants allocated time between the two stacks of problems in approximate proportion to the rates of reinforcement obtained from each alternative (i.e., 4:1). However, in subsequent experimental phases, pairing more difficult problems with the richer schedule of reinforcement failed to disrupt the allocation patterns produced by differing rates of reinforcement alone. Thus, for these individuals in this experimental context, manipulating problem difficulty did not affect time-allocation patterns produced by rate of reinforcement. Study 2 examined the effects reinforcer quality would produce in combination with different reinforcement rates and different problem difficulty. Unlike problem difficulty, reinforcer quality substantially disrupted the baseline allocation patterns produced by rate of reinforcement for both students. This biasing effect of reinforcer quality held even when combined with more difficult problems and one fourth the rate of reinforcement available for the alternative response. These findings have implications for understanding the effects of problem difficulty and reinforcer quality on socially relevant human behavior and for the study of behavioral allocation patterns in applied work within a matching theory framework.

Several applied studies have shown that the difficulty of a task is an important variable affecting task performance and maladaptive behavior correlated with task demands (e.g., Carr & Newsom, 1985; Cooper et al., 1990; Horner & Day, 1991; Mace et al., 1987; Weeks & Gaylord-Ross, 1981). These findings contrast with those of the present study, which showed that, compared to rate and quality of reinforcement, problem difficulty had little effect on how individuals allocated their responses across concurrent alternatives. Four factors might account for this discrepancy.

First, the magnitude of differences in response requirements for the two sets of arithmetic problems may have been too small to produce an effect in the present study. Although we acknowledge this possibility, the problem sets were chosen on the basis of the recommendation of the classroom teacher and on standardized test scores indicating the students' current level of mathematics achievement. In this instance, raising the difficulty level of problems further would have consisted of problems whose solution may have been beyond the repertoire of the subjects.

A second factor to consider is reinforcer potency. Although the subjects in this study were identified as having learning and behavior problems, both students in Study 1 reported high interest in earning money via participation in the study. Because money was contingent on completion of arithmetic problems, increasing the difficulty of problems correlated with the richer reinforcement schedule appears to have been insufficient to discount the monetary gains derived from that schedule. Perhaps biasing effects of problem difficulty would have emerged using a less potent reinforcer. In at least one basic research study, large rewards were found to override the separate effects of high-effort response requirements and punishment (Eisenberger, Weier, Masterson, & Theis, 1989).

A third consideration is that participants in research on the effects of problem difficulty were selectively targeted because their behavior problems cooccurred with the presentation of difficult tasks (e.g., Carr et al., 1980; Mace et al., 1987; Weeks & Gaylord-Ross, 1981). Subjects in these studies may have been especially sensitive to problem difficulty. In addition, the maladaptive behavior of subjects in many of these studies was maintained by escape or avoidance of tasks. This meant that qualitatively different reinforcers were available for the concurrent alternatives: escape for maladaptive behavior and positive reinforcement for task engagement. Under these conditions, reinforcer quality may have played an influential role (Neef et al., 1993).

A final consideration is the different procedures used to study the effects of problem difficulty on operant behavior. In a single-alternative situation, experimenters study the effects of varied difficult conditions on a single response alternative. For example, response force may be increased to study its suppressive effect on the absolute rate of responding (e.g., Chung, 1965). Thus, in a single-alternative situation, absolute values of problem difficulty may be varied, and their effects on absolute rate of responding are the subject of study.

In contrast, the effects of *relative* problem difficulty on relative response frequency are examined in the concurrent alternatives situation (Davison & Ferguson, 1978). In this procedure, two or more concurrently available response alternatives are specified. Experimenters can then vary the response requirements for each alternative and determine how relative differences in problem difficulty affect the relative allocation of behavior across the alternatives. The effects of problem difficulty and other choice-influencing variables are likely to differ depending on the reinforcement and response parameters of the concurrent alternatives. In the present study, the alternative with high problem difficulty was juxtaposed against an alternative with low difficulty and low-rate and low-quality reinforcement. Under these conditions, problem difficulty had little effect on time allocation. Thus, differences in the effects of problem difficulty on operant behavior may depend on the use of either a single alternative or concurrent alternatives.

The present findings also raise questions about the concept of "problem difficulty." In both studies, the difficulty of arithmetic problems was systematically varied under the assumption that some operations are more complex than others (e.g., addition vs. multiplication), as are problems containing more rather than fewer digits. Although this reasoning underlies most curriculum sequences and has ample precedence for affecting some types of social behavior (e.g., Cooper et al., 1990; Horner & Day, 1991; Mace et al., 1987; Weeks & Gaylord-Ross, 1981), it may be that once a skill is acquired, problem difficulty is less influential than the reinforcer dimensions of quality, rate, and delay (Neef et al., 1992, 1993). Another possibility to consider is that, for some individuals, increased problem difficulty may not equate to a decreased likelihood of the response. There is some evidence to suggest that preference and task difficulty follow a curvilinear function, with tasks of moderate difficulty being associated with greater preference than easy or very difficult ones (Harter, 1978). Thus, under some contexts, increased problem difficulty may function as a positive reinforcer (e.g., Ivana's preference for the combination of high reinforcer quality and high difficulty in Study 2).

Finally, there is growing evidence that human choice in applied situations is multiply determined by the reinforcer dimensions of rate, quality, and delay. Although the present study failed to demonstrate direct effects of problem difficulty on behavioral allocation, we believe the variable deserves further attention in choice research, especially with individuals who demonstrate difficulty in task situations. The fact that human choice is multiply determined suggests that a choice assessment paradigm is needed to assess the influences reinforcer and response dimensions have on a given individual's response-allocation patterns.

REFERENCES

Carr, E. G., & Durand, M. (1985). Reducing behavior problems through functional communication training. *Journal of Applied Behavior Analysis, 18,* 111–126.

Carr, E. G., & Newsom, C. (1985). Demand-related tantrums. *Behavior Modification, 9,* 403–426.

Carr, E. G., Newsom, C., & Binkoff, J. (1980). Escape as a factor in the aggressive behavior of two retarded children. *Journal of Applied Behavior Analysis, 13,* 101–117.

Chung, S. (1965). Effects of effort on response rate. *Journal of the Experimental Analysis of Behavior, 8,* 1–7.

Conger, R., & Killeen, P. (1974). Use of concurrent operants in small group research. *Pacific Sociological Review, 17,* 399–416.

Cooper, L. J., Wacker, D. P., Sasso, G. M., Reimers, T. M., & Donn, L. K. (1990). Using parents as therapists to evaluate the appropriate behavior of their children: Application to a tertiary diagnostic clinic. *Journal of Applied Behavior Analysis, 23,* 285–296.

Davison, M., & Ferguson, A. (1978). The effects of different component response requirements in multiple and concurrent schedules. *Journal of the Experimental Analysis of Behavior, 29,* 283–295.

Davison, M., & McCarthy, D. (1988). *The matching law: A research review.* Hillsdale, NJ: Erlbaum.

de Villiers, P. A. (1977). Choice in concurrent schedules and a quantitative formulation of the law of effect. In W. K. Honig & J. E. R. Staddon (Eds.), *Handbook of operant behavior* (pp. 233–287). New York: Prentice Hall.

Eisenberger, R., Weier, F., Masterson, F. A., & Theis, L. Y. (1989). Fixed-ratio schedules increase generalized self-control: Preference for large rewards despite high effort or punishment. *Journal of Experimental Psychology: Animal Behavior Processes, 15,* 383–392.

Friman, P. C., Finney, J. W., Rapoff, M. A., & Christophersen, E. R. (1985). Improving pediatric appointment-keeping with reminders and reduced response requirements. *Journal of Applied Behavior Analysis, 18,* 315–321.

Fuqua, R. W. (1984). Comments on the applied relevance of the matching law. *Journal of Applied Behavior Analysis, 17,* 381–386.

Harter, S. (1978). Pleasure derived from challenge and the effects of receiving grades on children's difficulty level choices. *Child Development, 49,* 788–799.

Herrnstein, R. J. (1961). Relative and absolute strength of a response as a function of frequency of reinforcement. *Journal of the Experimental Analysis of Behavior, 4,* 267–272.

Herrnstein, R. J. (1970). On the law of effect. *Journal of the Experimental Analysis of Behavior, 4,* 243–266.

Horner, R. H., & Day, H. M. (1991). The effects of response efficiency on functionally equivalent competing behaviors. *Journal of Applied Behavior Analysis, 24,* 719–732.

Hunter, I., & Davison, M. (1982). Independence of response force and reinforcement rate on concurrent variable-interval schedule performance. *Journal of the Experimental Analysis of Behavior, 37,* 183–197.

Mace, F. C., Browder, D., & Lin, Y. (1987). Analysis of demand conditions associated with stereotypy. *Journal of Behavior Therapy and Experimental Psychiatry, 18,* 25–31.

Mace, F. C., Neef, N. A., Shade, D., & Mauro, B. C. (1994). Limited matching on concurrent-schedule reinforcement of academic behavior. *Journal of Applied Behavior Analysis, 27,* 585–596.

Martens, B. K., & Houk, J. L. (1989). The application of Herrnstein's law of effect to disruptive and on-task behavior of a retarded adolescent girl. *Journal of the Experimental Analysis of Behavior, 51,* 17–28.

Martens, B. K., Lochner, D. G., & Kelly, S. Q. (1992). The effects of variable-interval reinforcement on academic engagement: A demonstration of matching theory. *Journal of Applied Behavior Analysis, 25,* 143–151.

McDowell, J. J. (1989). Two modern developments in matching theory. *The Behavior Analyst, 12,* 153–166.

Neef, N. A., Mace, F. C., & Shade, D. (1993). Impulsivity in students with serious emotional disturbance: The interactive effects of reinforcer rate, delay and quality. *Journal of Applied Behavior Analysis, 26,* 37–52.

Neef, N. A., Mace, F. C., Shea, M. C., & Shade, D. (1992). Effects of reinforcer rate and reinforcer quality on time allocation: Extensions of matching theory to educational settings. *Journal of Applied Behavior Analysis, 25,* 691–699.

Neef, N. A., Shade, D., & Miller, M. S. (1994). Assessing influential dimensions of reinforcers on choice in students with serious emotional disturbance. *Journal of Applied Behavior Analysis, 27,* 575–583.

Pierce, W. D., & Epling, W. F. (1983). Choice, matching, and human behavior: A review of the literature. *The Behavior Analyst, 6,* 57–76.

Weeks, M., & Gaylord-Ross, R. (1981). Task difficulty and aberrant behavior in severely handicapped students. *Journal of Applied Behavior Analysis, 14,* 449–463.

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APPENDIX

Measures of Fluency (Response Rate) and Accuracy (Number of Errors per Session) During the Equal-Difficulty Conditions for Ivana and Joyce (Study 1)

		Problem difficulty	Response rate		Errors per session	
Subject	Condition	Left stack/right stack	Yellow	Goldenrod	Yellow	Goldenrod
Ivana	Equal Difficulty A1	Low/low (10)	9.61	2.08	0.40	0.10
		High/high (10)	3.80	0.45	0.70	0.20
	Equal Difficulty A2	Low/low (15)	8.83	3.05	0.47	0.13
		High/high (15)	2.65	0.75	0.67	0.40
	Equal Difficulty A3	Low/low (8)	8.24	3.34	0.38	0
		High/high (8)	1.58	0.61	1.00	0.25
	Equal Difficulty A3	Low/low (5)	8.48	1.74	0.40	0
		High/high (5)	2.08	0.48	0.80	0.20
	Equal Difficulty A4	Low/low (3)	8.87	1.63	0.40	0
		High/high (3)	1.20	0.13	0.67	0
Joyce	Equal Difficulty A1	Low/low (3)	8.77	4.67	0.33	0.33
		High/high (3)	5.30	2.10	1.00	0.67
	Equal Difficulty A2	Low/low (4)	10.00	3.48	0	0
		High/high (4)	4.35	1.92	0	0
	Equal Difficulty A3	Low/low (4)	11.82	3.60	0.25	0.25
		High/high (4)	1.85	0.75	1.00	0.50

Note. The values shown in parentheses are the number of sessions. The entries for response rate and number of errors are the mean values calculated across the low- and high-difficulty procedures as they alternated across each of the equal-difficulty conditions.